Randall-Sundrum two D-brane model

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In Randall-Sundrum two D-brane system we derive the gravitational theory on the branes. It is turned out from the consistency that one D-brane has the negative tension brane under Randall-Sundrum tuning and both gauge fields on the brane are related by scale transformation through the bulk RR/NS-NS fields. As same with the single D-brane case, the gauge field which is supposed to be localised on the brane does not couple to the gravity on the branes.

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I. INTRODUCTION

Superstring theory provides us new picture of our universe in higher dimensional spacetimes: our universe may be a thin domain wall in higher dimensions. The simplest models were proposed by Randall and Sundrum [1, 2]. Therein the bulk spacetime and the brane are governed by the five dimensional Einstein gravity and the Nambu-Goto action, respectively. Although such pictures rely on the feature of D-brane, realistic models based on D-brane has not been seriously considered so far. Recently a single D-brane model which is the D-brane version of Randall-Sundrum II model [2] was addressed by long wave approximation in Ref. [3, 4]. Surprisingly, the gauge field which is supposed to localise on the brane cannot be source for the gravity on the brane when the net cosmological constant on the brane vanishes. If the net cosmological constant exists, the gauge fields can be source for the gravity [5]. However, there were undetermined terms which will be fixed by the boundary/initial condition near the Cauchy horizons in anti-deSitter spacetime. In general there is no guarantee that we can ignore such undetermined terms. Hence we will consider two D-brane system in this brief report. In this case the undetermined terms can be completely fixed by two junction conditions on the branes.

The rest of this paper is organised as follows. In Sec. II, we describe the setup of model, field equations and junction conditions. In Sec. III, we solve the bulk spacetime using long wave approximation [7] and derive the field equations on the brane. Finally we give summary and discussion in Sec. IV.

II. MODEL

A. The action for toy model

For simplicity we work with a toy model. It is a mimic of a theory derived from ten-dimensional type IIB supergravity through dimensional reduction to five. The main differences between the original one [3, 6] and the current toy model are as follows. In the current model, there are no scalar fields corresponding to dilaton and radius of compactification on S^5. Instead, we introduced the bulk cosmological constant Λ. In single D-brane case it is turned out that the contribution from scalar fields are not so important when one wants to discuss the coupling of the gauge field on the brane to the gravity. In fact, we will be able to see the same result, that is, the gauge field localised on the brane does not couple to the gravity. The cancellation between the contribution from the NS-NS and RR fields is essential. See Ref. [3, 6] for the original five dimensional action.

Now we are thinking of two D-brane model and its total action is given by

$$\begin{aligned} S \ = \ & \frac{1}{2\kappa^2}\int d^5x\sqrt{-G}\Big[{}^{(5)}R-2\Lambda-\frac{1}{2}|H|^2 \\ & -\frac{1}{2}(\nabla\chi)^2-\frac{1}{2}|\tilde{F}|^2-\frac{1}{2}|\tilde{G}|^2\Big] \\ & +S^{(+)}_{\rm brane}+S^{(+)}_{\rm CS}+S^{(-)}_{\rm brane}+S^{(-)}_{\rm CS}, \end{aligned} \tag{1}$$

where $H_{MNK} = \frac{1}{2}\partial_{[M}B_{NK]}$, $F_{MNK} = \frac{1}{2}\partial_{[M}C_{NK]}$, $G_{K_1K_2K_3K_4K_5} = \frac{1}{4!}\partial_{[K_1}D_{K_2K_3K_4K_5]}$, $\tilde{F} = F+\chi H$ and $\tilde{G}=G+C\wedge H$. $M,N,K=0,1,2,3,4$. B_{MN} and C_{MN} are 2-form fields, and $D_{K_1K_2K_3K_4}$ is a 4-form field. χ is a scalar field.

$S^{(\pm)}_{\rm brane}$ is given by Born-Infeld action

$$S^{(+)}_{\rm brane}=\gamma_{(+)}\int d^4x\sqrt{-\det(h+\mathcal{F}^{(+)})}, \tag{2}$$

$$S^{(-)}_{\rm brane}=\gamma_{(-)}\int d^4x\sqrt{-\det(q+\mathcal{F}^{(-)})}, \tag{3}$$

where $h_{\mu\nu}$ and $q_{\mu\nu}$ are the induced metric on the $D_\pm$-brane and

$$\mathcal{F}^{(\pm)}_{\mu\nu}=B^{(\pm)}_{\mu\nu}+(-\gamma_{(\pm)})^{-1/2}F^{(\pm)}_{\mu\nu}. \tag{4}$$

$F_{\mu\nu}$ is the $U(1)$ gauge field on the brane. $\mu,\nu=0,1,2,3$. $S^{(\pm)}_{\rm CS}$ is Chern-Simons action

$$S^{(+)}_{\rm CS} \ = \ \gamma_{(+)}\int d^4x\sqrt{-h}\epsilon^{\mu\nu\rho\sigma}\Big[\frac{1}{4}\mathcal{F}^{(+)}_{\mu\nu}C^{(+)}_{\rho\sigma}+\frac{\chi}{8}\mathcal{F}^{(+)}_{\mu\nu}\mathcal{F}^{(+)}_{\rho\sigma}$$

$$+\frac{1}{24}D^{(+)}_{\mu\nu\rho\sigma}\Bigg], \tag{5}$$

$$S^{(-)}_{\rm CS} = \gamma_{(-)}\int d^4x\sqrt{-q}\epsilon^{\mu\nu\rho\sigma}\Bigg[\frac{1}{4}\mathcal{F}^{(-)}_{\mu\nu}C^{(-)}_{\rho\sigma}+\frac{\chi}{8}\mathcal{F}^{(-)}_{\mu\nu}\mathcal{F}^{(-)}_{\rho\sigma} +\frac{1}{24}D^{(-)}_{\mu\nu\rho\sigma}\Bigg]. \tag{6}$$

B. Basic equations

In this subsection we write down the basic equations and boundary conditions. Let us perform (1+4)-decomposition

$$ds^2 = G_{AB}dx^Adx^B = e^{2\phi(x)}dy^2 + g_{\mu\nu}(y,x)dx^\mu dx^\nu, \tag{7}$$

where y is the coordinate orthogonal to the brane and $\mu,\nu = 0,1,2,3$. D_+-brane and D_--brane are supposed to locate at $y=0$ and $y=y_0$. For simplicity, we assume $\tilde{F}_{\mu\nu\alpha}=0$ and $H_{\mu\nu\alpha}=0$, that is, they are closed.

The "evolutional" equations to the y-direction are

$$e^{-\phi}\partial_y K = {}^{(4)}R - \kappa^2\left({}^{(5)}T^\mu_\mu - \frac{4}{3}{}^{(5)}T^M_M\right) - K^2 - e^{-\phi}D^2e^\phi, \tag{8}$$

$$e^{-\phi}\partial_y\tilde{K}^\mu_\nu = {}^{(4)}\tilde{R}^\mu_\nu - \kappa^2\left({}^{(5)}T^\mu_\nu - \frac{1}{4}\delta^\mu_\nu {}^{(5)}T^\alpha_\alpha\right) - K\tilde{K}^\mu_\nu - e^{-\phi}[D^\mu D_\nu e^\phi]_{\rm traceless}, \tag{9}$$

$$\partial_y^2\chi + D^2\chi + e^\phi K\partial_y\chi - \frac{1}{2}H_{y\alpha\beta}\tilde{F}^{y\alpha\beta} = 0, \tag{10}$$

$$\partial_y X^{y\mu\nu} + e^\phi KX^{y\mu\nu} + \frac{1}{2}F_{y\alpha\beta}\tilde{G}^{y\alpha\beta\mu\nu} = 0, \tag{11}$$

$$\partial_y\tilde{F}^{y\mu\nu} + e^\phi K\tilde{F}^{y\mu\nu} - \frac{1}{2}H_{y\alpha\beta}\tilde{G}^{y\alpha\beta\mu\nu} = 0, \tag{12}$$

$$\partial_y\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = e^\phi K\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{13}$$

where $X^{y\mu\nu} := H^{y\mu\nu} + \chi\tilde{F}^{y\mu\nu}$ and the energy-momentum tensor is

$$\begin{aligned}\kappa^2\,{}^{(5)}T_{MN} = {} & \frac{1}{2}\left[\nabla_M\chi\nabla_N\chi - \frac{1}{2}g_{MN}(\nabla\chi)^2\right] \\ & + \frac{1}{4}\left[H_{MKL}H_N{}^{KL} - g_{MN}|H|^2\right] \\ & + \frac{1}{4}\left[\tilde{F}_{MKL}\tilde{F}_N{}^{KL} - g_{MN}|\tilde{F}|^2\right] \\ & + \frac{1}{96}\tilde{G}_{MK_1K_2K_3K_4}\tilde{G}_N{}^{K_1K_2K_3K_4} - \Lambda g_{MN}.\end{aligned} \tag{14}$$

$K_{\mu\nu}$ is the extrinsic curvature, $K_{\mu\nu} = \frac{1}{2}e^{-\phi}\partial_y g_{\mu\nu}$. $\tilde{K}^\mu_\nu$ and ${}^{(4)}\tilde{R}^\mu_\nu$ are the traceless parts of K^μ_ν and ${}^{(4)}R^\mu_\nu$, respectively.

The constraints are

$$-\frac{1}{2}\left[{}^{(4)}R - \frac{3}{4}K^2 + \tilde{K}^\mu_\nu\tilde{K}^\nu_\mu\right] = \kappa^2\,{}^{(5)}T_{yy}e^{-2\phi}, \tag{15}$$

$$D_\nu K^\nu_\mu - D_\mu K = \kappa^2\,{}^{(5)}T_{\mu y}e^{-\phi}, \tag{16}$$

$$D^\alpha(e^{-\phi}X_{y\alpha\mu}) = 0, \tag{17}$$

$$D^\alpha(e^{-\phi}\tilde{F}_{y\alpha\mu}) = 0, \tag{18}$$

$$D^\alpha(e^{-\phi}\tilde{G}_{y\alpha\mu_1\mu_2\mu_3}) = 0, \tag{19}$$

where D_μ is the covariant derivative with respect to $g_{\mu\nu}$.

Under Z_2-symmetry, the junction conditions at the brane located $y=0$ are

$$K_{\mu\nu} - h_{\mu\nu}K = \frac{-\kappa^2\gamma_{(+)}}{2}(h_{\mu\nu} - T^{(+)}_{\mu\nu}) + O(T^2_{\mu\nu}) \tag{20}$$

$$H_{y\mu\nu}(0,x) = -\kappa^2\gamma_{(+)}e^\phi\mathcal{F}^{(+)}_{\mu\nu}, \tag{21}$$

$$\tilde{F}_{y\mu\nu}(0,x) = -\frac{\kappa^2}{2}\gamma_{(+)}e^\phi\epsilon_{\mu\nu\alpha\beta}\mathcal{F}^{(+)\alpha\beta}, \tag{22}$$

$$\tilde{G}_{y\mu\nu\alpha\beta}(0,x) = -\kappa^2\gamma_{(+)}e^\phi\epsilon_{\mu\nu\alpha\beta}, \tag{23}$$

$$\partial_y\chi(0,x) = -\frac{\kappa^2}{8}\gamma_{(+)}e^\phi\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{(+)}_{\mu\nu}\mathcal{F}^{(+)}_{\alpha\beta}. \tag{24}$$

The junction conditions at the brane located $y=y_0$ are

$$K_{\mu\nu} - q_{\mu\nu}K = \frac{\kappa^2\gamma_{(-)}}{2}(q_{\mu\nu} - T^{(-)}_{\mu\nu}) + O(T^2_{\mu\nu}) \tag{25}$$

$$H_{y\mu\nu}(y_0,x) = \kappa^2\gamma_{(-)}e^\phi\mathcal{F}^{(-)}_{\mu\nu}, \tag{26}$$

$$\tilde{F}_{y\mu\nu}(y_0,x) = \frac{\kappa^2}{2}\gamma_{(-)}e^\phi\epsilon_{\mu\nu\alpha\beta}\mathcal{F}^{(-)\alpha\beta}, \tag{27}$$

$$\tilde{G}_{y\mu\nu\alpha\beta}(y_0,x) = \kappa^2\gamma_{(-)}e^\phi\epsilon_{\mu\nu\alpha\beta}, \tag{28}$$

$$\partial_y\chi(y_0,x) = \frac{\kappa^2}{8}\gamma_{(-)}e^\phi\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{(-)}_{\mu\nu}\mathcal{F}^{(-)}_{\alpha\beta}. \tag{29}$$

In the above

$$T^{(\pm)\mu}{}_\nu = \mathcal{F}^{(\pm)\mu\alpha}\mathcal{F}^{(\pm)}_{\alpha\nu} - \frac{1}{4}\delta^\mu_\nu\mathcal{F}^{(\pm)}_{\alpha\beta}\mathcal{F}^{(\pm)\alpha\beta}. \tag{30}$$

III. LONG-WAVE APPROXIMATION

In this section, we approximately solve the bulk field equations by long wave approximation(gradient expansion [7]).

The bulk metric is written again as,

$$ds^2 = e^{2\phi(x)}dy^2 + g_{\mu\nu}(y,x)dx^\mu dx^\nu. \tag{31}$$

The induced metric on the brane will be denoted by $h_{\mu\nu} := g_{\mu\nu}(0,x)$ and then

$$g_{\mu\nu}(y,x) = a^2(y,x)\Big[h_{\mu\nu}(x) + \overset{(1)}{g}_{\mu\nu}(y,x) + \cdots\Big]. \quad (32)$$

In the above $\overset{(1)}{g}_{\mu\nu}(0,x) = 0$ and $a(0,x) = 1$. In a similar way, the extrinsic curvature is expanded as

$$K^\mu_\nu = \overset{(0)}{K^\mu_\nu} + \overset{(1)}{K^\mu_\nu} + \overset{(2)}{K^\mu_\nu} + \cdots. \quad (33)$$

The small parameter is $\epsilon = (\ell/L)^2 \ll 1$, where L and ℓ are the curvature scale on the brane and the bulk anti-deSitter curvature scale, respectively.

A. 0th order

It is easy to obtain the zeroth order solutions. Without derivation we present them.

$$\overset{(0)}{K^\mu_\nu} = -\frac{1}{\ell}\delta^\mu_\nu, \quad (34)$$

$$\overset{(0)}{g}_{\mu\nu} = a^2(y,x)h_{\mu\nu}(x) = e^{-\frac{2d(y,x)}{\ell}}h_{\mu\nu}(x), \quad (35)$$

where

$$d(y,x) = \int_0^y dy e^{\phi(x)}, \quad (36)$$

$$\frac{1}{\ell} = -\frac{1}{6}\kappa^2\gamma_{(+)} = \frac{1}{6}\kappa^2\gamma_{(-)} := -\frac{1}{6}\kappa^2\gamma, \quad (37)$$

and

$$2\Lambda + \frac{5\kappa^4}{6}\gamma^2 = 0. \quad (38)$$

ℓ is the curvature scale of anti-deSitter like spacetimes. Eqs. (37) and (38) represent the Randall-Sundrum tuning and then the tension $\gamma_{(+)}$ and $\gamma_{(-)}$ have the same magnitude with opposite signature, $\gamma_{(+)} = -\gamma_{(-)} < 0$. In this tuning, the brane geometry could be four dimensional Minkowski spacetime.

In addition,

$$\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = -a^4\kappa^2\gamma e^\phi \epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}, \quad (39)$$

where $\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}$ is the Levi-Civita tensor with respect to the induced metric $h_{\mu\nu}$ on the brane.

B. 1st order

Since $\chi = O(\mathcal{F}^2)$ and the contribution of χ to gravitational field equation is appeared as form $(\nabla\chi)^2 = O(\mathcal{F}^4)$, we will omit such terms which will be important in the next order.

The first order equations for $\tilde{F}_{y\mu\nu}$ and $H_{y\mu\nu}$ are

$$\partial_y \overset{(1)}{\tilde{F}}_{y\mu\nu} - \frac{1}{2a^4}\overset{(1)}{H}_{y\alpha\beta}\tilde{G}_{y\rho\sigma\mu\nu}h^{\alpha\rho}h^{\beta\sigma} = 0, \quad (40)$$

and

$$\partial_y \overset{(1)}{H}_{y\mu\nu} + \frac{1}{2a^4}\overset{(1)}{\tilde{F}}_{y\alpha\beta}\tilde{G}_{y\rho\sigma\mu\nu}h^{\alpha\rho}h^{\beta\sigma} = 0. \quad (41)$$

Together with the junction conditions on D_+-brane the solutions are given by

$$\overset{(1)}{H}_{y\mu\nu}(y,x) = -\kappa^2\gamma a^{-6}e^\phi\mathcal{F}^{(+)}_{\mu\nu}, \quad (42)$$

and

$$\overset{(1)}{\tilde{F}}_{y\mu\nu}(y,x) = -\frac{\kappa^2}{2}\gamma a^{-6}e^\phi\epsilon_{\mu\nu\rho\sigma}\mathcal{F}^{(+)}_{\alpha\beta}h^{\rho\alpha}h^{\sigma\beta}. \quad (43)$$

The remaining junction conditions on D_--brane imply the relation between $\mathcal{F}^{(+)}_{\mu\nu}$ and $\mathcal{F}^{(-)}_{\mu\nu}$ as

$$\mathcal{F}^{(-)}_{\mu\nu} = a_0^{-6}\mathcal{F}^{(+)}_{\mu\nu}, \quad (44)$$

and then

$$T^{(-)}_{\mu\nu} = a_0^{-14}T^{(+)}_{\mu\nu}, \quad (45)$$

where $a_0 = a(y_0,x) = e^{-d_0(x)/\ell}$ and $d_0(x) := d(y_0,x)$.

Let us first substitute the junction conditions for $H_{y\mu\nu}$ and $\tilde{F}_{y\mu\nu}$ on the D_+ brane into the constraint equations of Eqs. (17) and (18). Then we see

$$\mathcal{D}^\mu\mathcal{F}^{(+)}_{\mu\nu} = 0, \quad (46)$$

$$\epsilon^{\mu\nu\alpha\beta}\mathcal{D}_\nu\mathcal{F}^{(+)}_{\alpha\beta} = 0, \quad (47)$$

where $\mathcal{D}_\mu$ is the covariant derivative with respect to $h_{\mu\nu}$.

Here we remind that the consistency of the assumptions of $H_{\mu\nu\alpha} = \tilde{F}_{\mu\nu\alpha} = 0$ implies ϕ =constant. To see this we begin with the identity

$$\epsilon^{\beta\alpha\mu\nu}D_\alpha\tilde{F}_{y\mu\nu} = 0, \quad (48)$$

which is easily derived from $\tilde{F}_{\mu\nu\alpha} = 0$ and the definition of $\tilde{F}_{MNK}$. Then we put the junction conditions on the D_+-brane into the above, that is,

$$\epsilon^{\beta\alpha\mu\nu}\mathcal{D}_\alpha(e^\phi\epsilon_{\mu\nu\rho\sigma}\mathcal{F}^{(+)\rho\sigma}) = 0. \quad (49)$$

Using Eq. (46) we can see that the above equation becomes

$$\mathcal{F}^{(+)\alpha\beta}\mathcal{D}_\alpha e^\phi = 0. \quad (50)$$

Therefore ϕ =const. is required under the assumption of $H_{\mu\nu\alpha} = \tilde{F}_{\mu\nu\alpha} = 0$. From now on we will set $\phi = 0$ without loss of generality.

Using these results the evolutional equation for the traceless part of the extrinsic curvature is

$$e^{-\phi}\partial_y \overset{(1)}{\tilde{K}^\mu_\nu} = -\overset{(0)}{K}\overset{(1)}{\tilde{K}^\mu_\nu} + \frac{{}^{(4)}\tilde{R}^\mu_\nu(h)}{a^2} - \kappa^4\gamma^2 a^{-16}T^{(+)\mu}{}_\nu, \quad (51)$$

where ${}^{(4)}R^\mu_\nu(h)=h^{\mu\alpha(4)}R_{\alpha\nu}(h)$ is the Ricci tensor with respect to $h_{\mu\nu}$ and $T^\mu_\nu=h^{\mu\alpha}T_{\alpha\nu}$.

The solution is summarised as

$$\overset{(1)}{\tilde{K}^\mu_\nu}(y,x)=-\frac{\ell^{(4)}\tilde{R}^\mu_\nu(h)}{2a^2}+\frac{\kappa^2\gamma}{2}a^{-16}T^{(+)\mu}_{\ \ \nu}+\frac{\chi^\mu_\nu(x)}{a^4}, \quad (52)$$

where χ^μ_ν is the "integration of constant". The solution to the trace part of the extrinsic curvature is

$$\overset{(1)}{K}(y,x)\;=\;-\frac{\ell}{6a^2}{}^{(4)}R(h). \quad (53)$$

Using the junction conditions on the D_+-brane Eqs (52) and (53) becomes

$${}^{(4)}\tilde{R}^\mu_\nu(h)=\frac{2}{\ell}\chi^\mu_\nu(x), \quad (54)$$

and

$$0=\overset{(1)}{K}(0,x)=-\frac{\ell}{6}{}^{(4)}R(h). \quad (55)$$

They correspond to the Einstein equation on the brane obtained in Ref. [8] and χ^μ_ν is projected Weyl tensor $E_{\mu\nu}$. For the moment, $\chi^\mu_\nu(x)$ is unknown term.

On D_--brane, Eq. (52) becomes

$$\frac{\kappa^2\gamma}{2}T^{(-)\mu}_{\ \ \nu}=-\frac{\ell^{(4)}\tilde{R}^\mu_\nu(h)}{2a_0^2}+\frac{\kappa^2\gamma}{2}a_0^{-16}T^{(+)\mu}_{\ \ \nu}+\frac{\chi^\mu_\nu(x)}{a_0^4}. \quad (56)$$

Using Eq. (45) the energy-momentum tensor in both sides are exactly canceled out and then

$$\chi^\mu_\nu(x)=\frac{\ell}{2}a_0^{2(4)}\tilde{R}^\mu_\nu(h). \quad (57)$$

All together we obtain the Einstein equation on D_+ brane

$$(1-a_0^2)^{(4)}G_{\mu\nu}(h)=0. \quad (58)$$

This is main result in our paper. The gauge fields do not appear in the right-hand side. However, there is the equation for the gauge field obtained from the constraint equations(See Eqs. (46) and (47)).

IV. SUMMARY AND DISCUSSION

We considered the two D-brane system under the Randall-Sundrum tuning. Therein U(1) gauge fields on the brane plays role as the source for the three form fields in the bulk. Then we derived the field equations on the brane after solving the bulk field equation with junction condition in the two brane system where the long wave approximation is valid. Consequently we find that the gravity on the brane does not couple to the gauge field. The result is same with the single D-brane case. Under the assumptions of $H_{\alpha\beta\mu}=\tilde{F}_{\alpha\beta\mu}=0$, the physical mode, radion, which expresses the distance between two branes is required to be constant. From our procedure, we can expect that the same result will be obtained for Yang-Mills field localised on the brane.

The result we obtained here is quite unusual. However, we do not know the situation where the effect of the gauge field to the gravity is important except for homogeneous and isotropic universe. In standard big-bang scenario the universe is dominated by the radiation before the equal time. The success of the big-bang scenario suggests the expansion of the universe is governed by unknown exotic matter like dark energy if one is serious about our result. On the other hand, we know the situation where the effect of the fermion to the gravity is important. If we see the similar result for fermions, it will be crisis in D-braneworld cosmology. There is a solution to this problem. As guessed in Ref. [5], the net cosmological constant on the brane might be important and its presence implies the coupling of the gravity to matter.

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